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Special Contribution

Numerical Simulation of Penetration into Ground

Yosuke Higo

Professor, Graduate School of Management (double appointment)
Graduate School of Engineering, Dept. Urban Management
Kyoto University

1. Introduction

When structures such as sheet piles and piles are driven into the ground, the ground around the structures is significantly disturbed by large shear forces due to friction between the ground and the structures. In the case of pile driving, large deformation also occurs at the pile tip. It is very important to accurately analyze such large deformation of the ground and to evaluate the impact of structure penetration on the surrounding ground. This article reviews the Material Point Method (MPM), which is one of the mesh-free deformation simulation methods for soils. MPM has been rapidly developed as a method for simulating large deformation of solids and is spreading internationally. Applicability of MPM to numerical simulation of penetration into ground is shown through numerical examples of sheet pile penetration and footing penetration simulations.

2. Material Point Method

Numerical simulation techniques within a continuum mechanics framework are classified into three primary groups based on their definition of the element control domain. The initial category is the mesh-based approach, represented by the Finite Element Method (FEM). In FEM, the element domain is determined by nodes and their connectivity, distinctly outlining the control domain through node coordinates. Though adept for boundary value problems, FEM can face mesh-tangling during severe deformation due to the connectivity of elements. The second category involves particle-based method, where material properties gather around representative points. Interpolation functions define the circular (2D) or spherical (3D) control domain shape. Well-known within particle-based methods are the Moving Particle Semi-implicit (MPS) method (e.g., Koshizuka and Oka, 1996) and Smooth Particle Hydro-dynamics (SPH) method (e.g., Monaghan and Lattanzio, 1985), which describe the material domain through particle assemblies. Unlike FEM, particle-based methods lack clear material boundaries, necessitating special treatment of boundaries. Conversely, particle-based methods can avoid mesh-tangling since individual particle domains, formed by interpolation, remain disconnected. The third type is the grid-particle hybrid method, typified by the Particle-in-Cell (PIC) method (Harlow, 1956), Fluid-Implicit-Particle (FLIP) method (Brackbill and Ruppel, 1986) for fluid mechanics, and Material Point Method (MPM) (Sulsky et al., 1994) for solid mechanics. These methods also depict the material domain through particle assemblies. While similar to other particle-based methods in simulating objects through arranged particles, grid-particle hybrids offer clear boundary condition definition by applying conditions to grids. The boundary setup in hybrid methods resembles FEM, facilitating easier configuration compared to particle-based approaches. In these methods, material properties center on material points, while governing equations are solved at grid nodes. Interpolation functions aid in exchanging physical quantities between material and grid nodes.

Sulsky et al. (1994) initially introduced MPM as a derivative of the PIC method for solid mechanics, termed the "original MPM" in this context to differentiate from other MPM derivatives. Ongoing MPM evolution mainly centers on integrating control domain deformation. A novel method applied to geomaterial deformation, termed Arbitrary Particle Domain Interpolation (APDI) permits flexible material point control domain definitions (Kiriyama and Higo 2020). Fig. 1 illustrates the relationships among various 2D and 3D MPM implementations, defining their control domains. The original MPM represents a material point as a simple point (Fig. 1(a)), but this leads to numerical oscillations when material points cross grid boundaries. To mitigate this, Bardenhagen and Kober (2004) proposed the Generalized Interpolation Material Point (GIMP) method. GIMP manifests in uniform/unchanged (uGIMP) and contiguous particle (cpGIMP) forms. In cpGIMP (Fig. 1(c)), the control domain updates with deformation, while Sadeghirad et al. (2011, 2013) further advanced GIMP into the Convected Particle Domain Interpolation (CPDI) method, accommodating shear and rotational control domain deformations. CPDI divides into CPDI1 (Fig. 1(d)), with parallelogram deformations, and CPDI2 (Fig. 1(e)), with quadrilateral deformations. Extending to three dimensions involves expanding from two dimensions for the original MPM, GIMP, and CPD1 methods. CPDI2's three-dimensional formulation necessitates a distinct approach, using direct integration for volume integration. The APDI method circumvents this complexity through numerical integration, offering a comprehensive formulation for both 2D and 3D, along with adaptable control domain deformations. CPDI2 uses quadrilateral control domain deformation, whereas APDI accommodates diverse control domain deformations.

The governing equations of MPM are fundamental laws of mechanics: the conservation of mass and the linear momentum. The mass conservation always holds because the material point possesses mass throughout the calculations. Hence the conservation of the linear momentum, i.e., the equation of motion, is only one equation to be solved. Note that governing equations for liquid phases and interaction with the solid phase, e.g., Darcy's law, are needed when simulating hydro-mechanical coupled behavior of soils as porous media. The algorithm of MPM is schematically shown in Fig. 2. The continuum body is divided into multiple subdomains and represented by material points with mass of the subdomain. The history-dependent variables such as stress and strain are stored and delivered by material points. Hence, history-dependent constitutive equations can be introduced. The governing equations are solved on the background Eulerian computational grid and thus MPM does not face to mesh tangling problem even when the severely large deformation. Similar to FEM, the boundary conditions are imposed on the computational grids. Although the grids containing the particles can roughly define the boundaries of a continuum body, exact representation of the boundaries need coupling with element-based method.

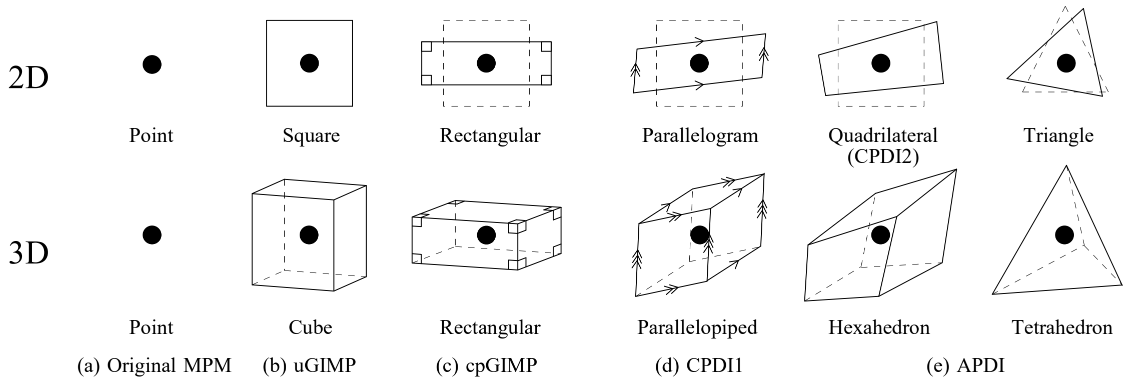


Fig. 1. Variations of particle domain for Material Point Method

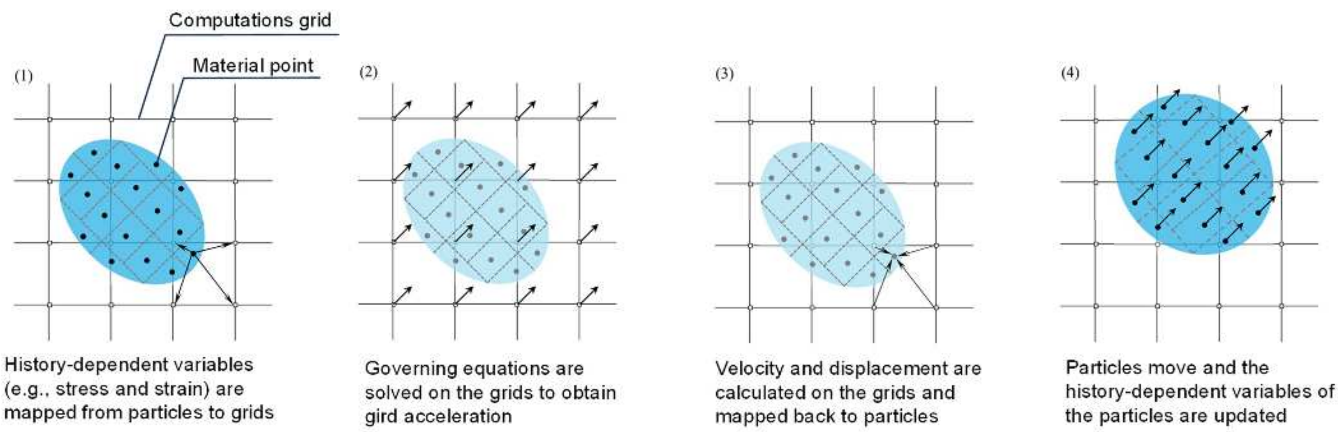


Fig. 2. Numerical algorithm of Material Point Method

3. Steel Sheet Pile Penetration

Numerical simulation of steel sheet pile penetration was conducted using a displacement-control method for simulating quasi-static press-in penetration and a stress control method for simulating dynamic hammer-blow penetration. The particle configuration and the boundary conditions are shown in Fig. 3. The thickness of the steel sheet pile is described by four particles allocated in two grids, which is much thicker than the real sheet piles, to avoid numerical instability when using smaller number of particles. The larger number of particles in one computational grid generally makes the accuracy of MPM simulation higher. The steel sheet pile is assumed to be a linear elastic body whose Lame constants λ and μ are 4.2×10^7 kN/m^2 are 2.80×10^7 kN/m^2, respectively. Although the second moment of area is larger than that of the real sheet pile, the set of elastic constants is determined to satisfy the bending stiffness of a real steel sheet pile. Consequently, the axial stiffness is underestimated. The model ground is an elasto-plastic body described by Mohr-Coulomb type yield function and Drucker-Pager type potential function. The internal friction angle and the cohesion are 30 degree and 8.5 kN/m^2, respectively. Particles of the ground in the vicinity of the sheet pile are modeled as interface particles whose internal friction angle and cohesion are assumed to be 20 degree and zero, respectively.

The loading conditions are shown in Fig. 3. In the displacement-controlled method, a constant displacement of 0.02 m/s was applied to the steel particles above the ground surface to penetrate the sheet pile. In the stress-control method, the top four steel particles were subjected to linearly varying surface forces in the form of triangular waves at intervals of 0.04 s to a maximum of 100 kN per unit depth, for a total of 400 kN, and the same surface forces were applied again after 0.56 s, repeated 19 times. The results of the analysis for a single blow were conducted beforehand, and the blow interval was determined as the time required for the ground vibration caused by the blows to settle down. The number of blows was set to be sufficient for the steel head to penetrate below the ground surface.

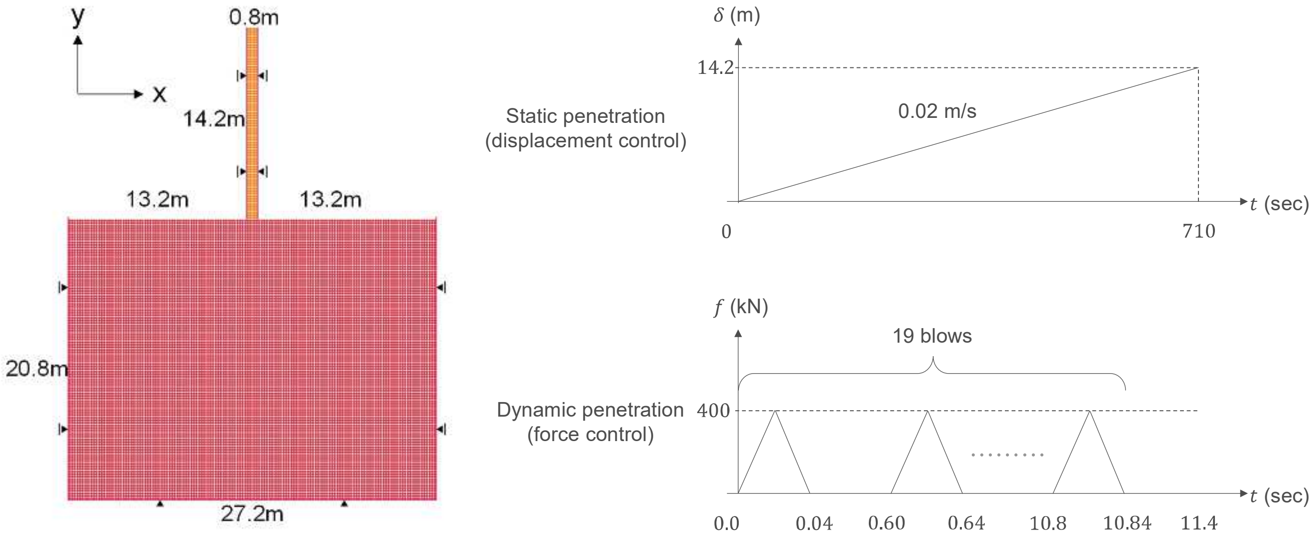
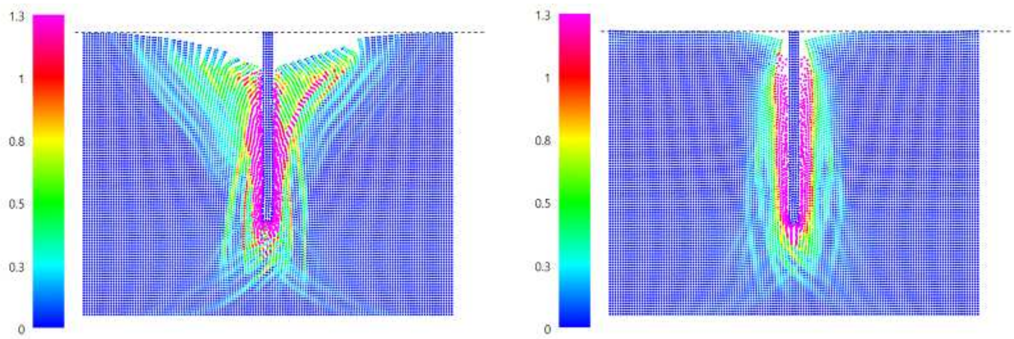
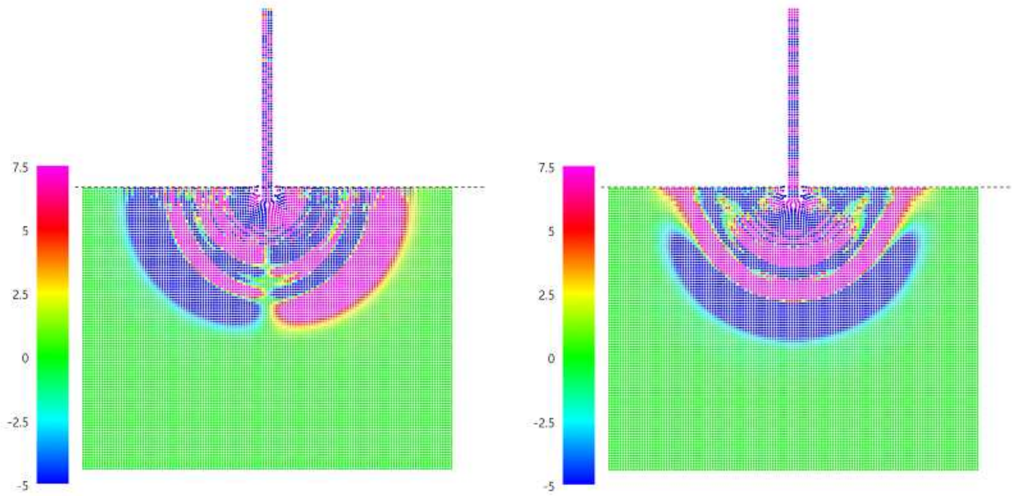


Fig. 3. 2D sheet pile penetration model (unit: m) and the loading conditions.



(a) Static press-in penetration (b) Dynamic hammer-blow penetration

Fig. 4. Numerical simulation of sheet pile penetration into ground. Maximum shear strain distribution for the static and dynamic penetration cases.



(a) x-direction (b) y-direction

Fig.5. Distribution of acceleration for the dynamic penetration case just after the first blow (0.12 second).

Fig. 4 show the maximum shear strain distribution at the end of penetration for the static and dynamic penetration. The results show that both the displacement-controlled and stress-controlled methods can simulate the behavior of steel penetration into the ground with large shear strain around the sheet pile. In the case of static penetration, the sheet pile drags the surrounding soil into the ground as the sheet pile penetrates, resulting in a large settlement. On the other hand, in the case of dynamic penetration, the steel penetration is observed without significant drag of the ground as observed in the case of the static press-in penetration.

The acceleration distributions in the x and y directions for the first blow in the dynamic penetration analysis are shown in Fig. 5. In dynamic penetration, the acceleration is distributed alternately positive and negative, and the acceleration propagates in the direction away from the sheet pile. In other words, the impact by the blow generates sparse and dense waves in the ground. While the ground is sparse near the steel, the confining pressure on the steel is reduced, which also reduces the frictional resistance between the ground and the sheet pile, which suppresses the dragging of the surrounding ground. On the other hand, in static penetration, the steel is constantly confined by the surrounding ground during penetration. Thus the frictional resistance is not reduced and the shear surface due to the frictional force reaches the ground surface as shown in Fig. 5. This causes significant drag of the surrounding ground over a wide area.

To evaluate even the stress level of sheet pile, much more particles must be placed in the thickness direction of the sheet pile, which is computationally too expensive. Although such an analysis is possible with a large-scale analysis using parallel computation, it will be difficult to implement in practice in the near future. However, it is possible to evaluate the behavior of the surrounding ground at least in practice by correctly evaluating the interaction between the ground and the structure.

4. Footing Penetration

Computational cost of three-dimensional numerical simulation by an explicit numerical integration scheme is too expensive to be done. A practical implementation scheme, the particle-element coupled method has been proposed. In this scheme, with the GIMP interpolation function, highly deformable regions are represented by material points. With the APDI interpolation function, material points are used, as if they were FEM elements, in the rest of the highly deformable regions. This combination of the GIMP and APDI methods makes it possible to divide the material domain into spatial discretization with different sizes. The particle-element is able to simulate extreme deformation by GIMP particles as well as to evaluate boundary values by APDI elements for the area of focus, which is sometimes difficult in purely particle-based methods due to the lack of an element boundary. The boundary between the particle and the element domains does not require any special treatments, but simply use interpolation function which satisfies partition of unity for each domain.

The three-dimensional model of the shallow foundation bearing capacity problem is shown in Fig. 6. The space around the footing and down to the bottom of the base ground is modeled with GIMP particles in a cubic manner. Each computational grid is a cube, 0.2 m on each side, and 8 particles are allocated per cell for the GIMP particles. Considering the symmetry of the scenario, a quarter of the domain is modeled. The normal direction of the lateral boundary is fixed (vertical roller condition), and the bottom surface is fixed. The other conditions for the three-dimensional analysis are listed in Table 1. Vertical load is incrementally applied to the base ground from 100 kPa to 600 kPa.

The snapshots of maximum shear strain distribution are shown in Fig. 7. At 100 kPa, the maximum shear strain is distributed like a stress bulb in the Boussinesq solution based on the elastic theory. At 200 kPa, the maximum shear strain is concentrated at the outer edge of the footing, forming a punching wedge beneath the footing. This strain concentration at the outer edge of the footing is identical to the theoretical solution for cohesive material. In the regime beyond 300 kPa, the base ground reaches the slip failure mode and the resisting slip line spreads in space. Settlement then increases continually to 400 kPa and surface deformation becomes visible in Fig. 7(d). However, there is no visual discontinuity in the base ground at this stage. The loading block continues to penetrate into the base ground up to 500 kPa and 600 kPa, where discontinuity between the loading block and the base ground becomes visible (Fig. 7(e) and (f)).

These results demonstrate that the particle-element coupled method is applicable to three-dimensional problems, reducing the implemental complexity and computational cost by dividing the domain into two parts (a large deformation domain and a small deformation domain). The computational cost is reduced by implementing the GIMP particles in the large deformed domain and the APDI particles in the small deformation domain. Use of a full particle model for the three-dimensional problem would result in an extremely high computational cost as compared with the two-dimensional model, but the particle-element coupled method makes it possible to perform the three-dimensional simulation at a reasonable computational cost.

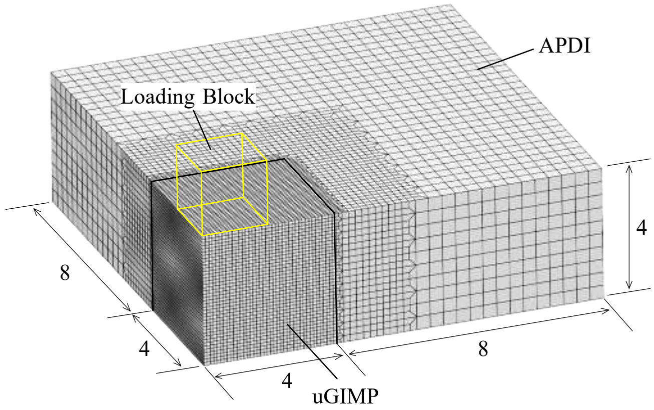
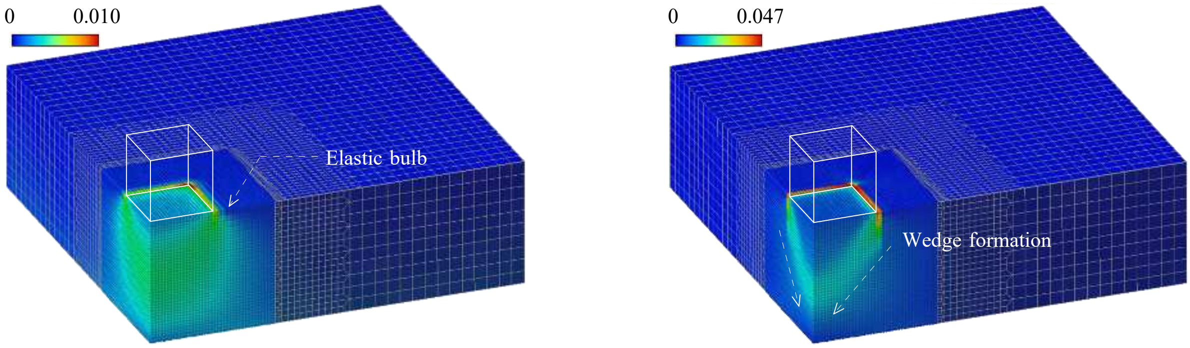


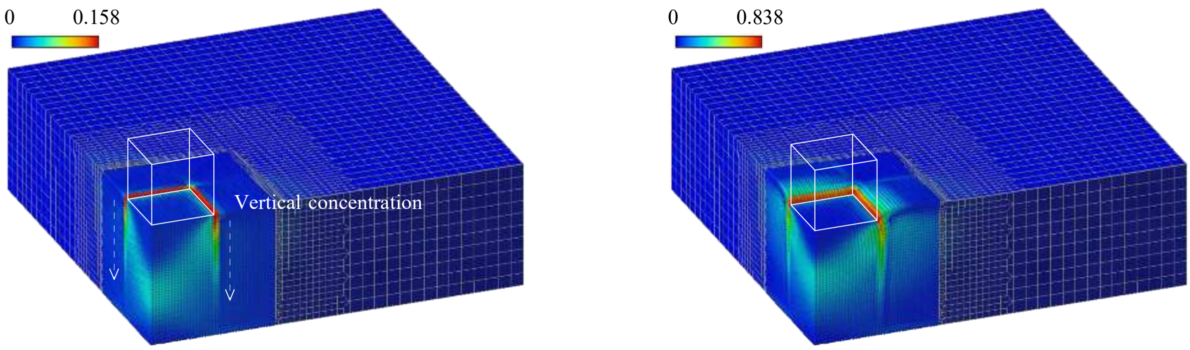
Fig. 6. 3D element-particle coupled footing penetration model (unit: m)

Items	Values
Particles per cell	8
Dimensions(H x W x D)	6 m x 12 m x 12 m
Width of cell	0.2 m
Time increment	0.00025
Damping factor	0.8
Convergence criteria	0.0001
Number of uGIMP interpolation Material Points	
Nearby Ground	64,000
Number of APDI interpolation Material Points	
Loading Block	8,000
Surrounding Ground	24,580

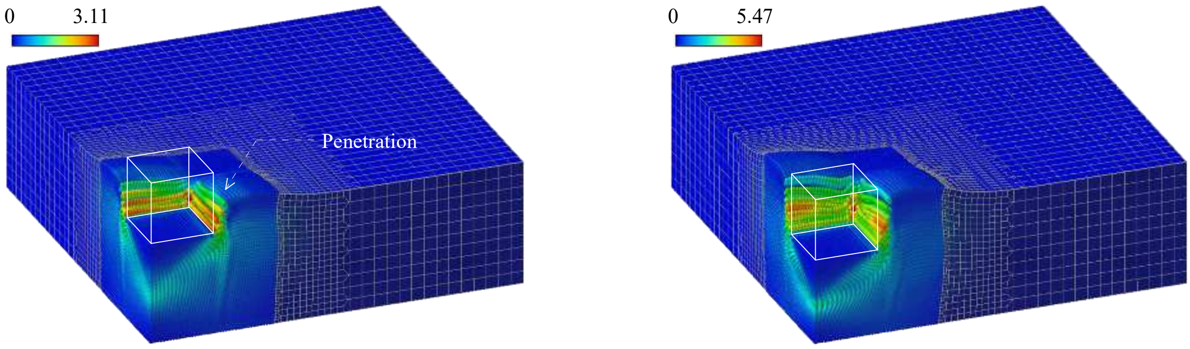
Table 1. Simulation conditions



(a) 100 (kN/m^2) (b) Pressure=200 (kN/m^2)



(c) Pressure=300 (kN/m^2) (d) Pressure=400 (kN/m^2)



(e) Pressure=500 (kN/m^2) (f) Pressure=600 (kN/m^2)

Fig. 7. 3D numerical simulation of footing penetration into ground. Maximum shear strain distribution at increasing

loading pressures.

5. Summary and Future Perspectives

The numerical simulations of penetration into ground successfully performed by MPM are demonstrated. To evaluate even the stress level of steel, multiple particles must be placed in the thickness direction of the steel, which requires a very large number of particles and is computationally too expensive. Although such an analysis is possible with a large-scale analysis using parallel computation, it will be difficult to implement in practice in the near future. However, it is possible to evaluate the behavior of the surrounding ground at least in practice by correctly evaluating the interaction between the ground and the structure.

In general, design and analysis are performed assuming ideal conditions after sheet pile and pile driving. In addition to MPM, several other methods have been developed that can be applied to large deformation simulations. Among them, MPM is similar to FEM in that the governing equations are solved on a background grid, and users of FEM, which is currently the most popular numerical analysis method, can use MPM relatively easily.

Acknowledgements

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