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Size effect of footing in ultimate bearing capacity of intermediate soil

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ABSTRACT: In geotechnical engineering, bearing capacity of rigid footings is perhaps common and complex problem in numerous facets. Although, a lot of researches have already been conducted pertaining to the bearing capacity but hardly a few researches present the size effect of foundation on ultimate bearing capacity (UBC) of intermediate soil using non-linear shear strength characteristics. The prime objective of this study is to analyze the effect of footing sizes on UBC by using the finite element analysis and assess the effectiveness of Architectural Institute of Japan's (AIJ) semi-empirical bearing capacity equation. Rigid plastic finite element method (RPFEM) using nonlinear shear strength characteristics of the soil is employed to evaluate the UBC of footing against the centric vertical load. The analysis results are compared with that of conventional bearing capacity formulae and a new bearing capacity equation is proposed with dimensional correction factor in cohesive part of AIJ's bearing capacity equation.

1 BACKGROUND

The precise estimation of the ultimate bearing capacity of foundation is the first and foremost step in order to ensure the stable footing-soil system. Its importance increases significantly especially in designing buildings or structures. A German engineer Ludwig Prandtl (1921) is globally admitted to be the torchbearer in development of primitive bearing capacity theory, where he utilized the theory of plasticity to understand the punching failure pattern of thick metals. The aforementioned researcher considered a weightless and infinite half-space just below the footing, to have strength characteristics "cohesion c" and "angle of internal friction ϕ" to illustrate the kinematic failure mode. The theory was further extended by Reissner (1924) who considered the perfectly frictional soil (c=0) loaded by adjacent uniform surcharge load. The closed form solution using the hyperbolic functions, resulted in the bearing capacity factor N_q. The ultimate bearing capacity equation developed by Terzaghi (1943) considering the effects of cohesion, material weight and surcharge load was a remarkable accomplishment in the concepts for laying the foundation of modern bearing capacity theories. The ultimate bearing capacity equation proposed by Terzaghi (1943) is as follows:

$$q = cN_c + 0.5\gamma BN_\gamma + \gamma D_f N_q \tag{1}$$

In the above equation; N_c, N_γ and N_q denote the soil bearing capacity factors. These factors depend upon the angle of internal friction of the material, ϕ. Other parameters given in the above equation are given below:

γ : Unit weight of soil (KN/m^3),
c : Soil cohesion (KN/m^2),
B : Width of foundation (m) and
D_f : Foundation depth (m)

A lot of researches have been conducted over the period of time for estimation of bearing capacity factors. By using the concept of moment equilibrium the boundary value solutions for N_q and N_c can be obtained, as proposed by Prandtl (1921) and Reissner (1924):

$$N_q = e^{\pi \tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right) \tag{2}$$

$$N_c = (N_q - 1)\cot\phi \tag{3}$$

Similarly, several researches have been conducted pertaining to the bearing capacity factor N_γ and relations are proposed accordingly. The research conducted

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by Meyerhof (1963) resulted in the following mathematical equation:

$$N_\gamma = (N_q - 1)\tan(1.4\phi) \tag{4}$$

The bearing capacity theory was further extended by Meyerhof (1963) by introducing depth and inclination factors for the situations where a line of action of applied load is inclined to the vertical plane. The following equation was proposed by the said researcher:

$$q = cN_c d_c i_c + 0.5\gamma B N_\gamma d_\gamma i_\gamma + \gamma D_f N_q d_q i_q \tag{5}$$

$$i_c = i_q = \left(1 - \frac{\theta^\circ}{90^\circ}\right)^2 \tag{6}$$

$$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2 \tag{7}$$

$$d_c = 1 + 0.2 \cdot \sqrt{k_p} \cdot \frac{D_f}{B} \tag{8}$$

$$d_q = d_\gamma = 1 + 0.1 \cdot \sqrt{k_p} \cdot \frac{D_f}{B} \tag{9}$$

Here in the above equations, θ represents the degree of inclined load with reference to the vertical axis. Coefficient of passive earth pressure is given as follows:

$$k_p = \tan^2\left(45 + \frac{\phi}{2}\right) \tag{10}$$

The Architectural Institute of Japan (AIJ, 1988, 2001) proposed the semi-experimentally devised ultimate bearing capacity formula. This equation is being used all across Japan for the ultimate bearing capacity estimation. The ultimate bearing capacity formula of AIJ in terms of bearing capacity factors N_c, N_γ and N_q can be written as follows:

$$q = i_c \alpha c N_c + i_\gamma \gamma \beta B \eta N_\gamma + i_q \gamma D_f N_q \tag{11}$$

Here, α and β denote the shape coefficients while η is the foundation size effect factor. De Beer (1970) used empirical or semi empirical techniques to propose shape modifiers i.e. α=1 and β=0.5. Relationship for the foundation size effect factor is expressed as follows:

$$\eta = \left(\frac{B}{B_o}\right)^m \tag{12}$$

$B_0 =$ 1m(Reference value in the footing width)

Based on the experimental considerations, m= -1/3 is recommended in engineering practices.

In contrast with the conventional bearing capacity formulations, AIJ equation considers the size effect of foundation on the ultimate bearing capacity. Therefore, the traditional approach results in the overvaluation of the calculated results, as the foundation width increased. The extent up to which foundation size influences the bearing capacity, needs to be carefully evaluated. The non-linear finite element method was also used by Ueno et al. (1998) to predict the confining stress dependence of material strength parameters i.e. "cohesion *c*" and "angle of internal friction ϕ" and consequently the shear failure criteria. Their research results showed that mean stress beneath the foundation varied from $2\gamma B$ to $10\gamma B$ in the case of strip footing and has considerable effect on material strength characteristics and hence the bearing capacity while taking into account the size effect. In this study primary focus is laid on evaluation of ultimate bearing capacity of foundation placed on intermediate soil subjected to centric vertical load. Moreover, this research also analyzes the size effect of footing in the bearing capacity for proposing the size effect factor in cohesive part of the AIJ bearing capacity formula. Rigid plastic finite element method (RPFEM) using the non-linear shear strength envelope against the confining stress is employed for the finite element analysis (FEA). Recently, use of finite element analysis is becoming increasingly common in almost all fields of engineering because of accuracy of obtained results and saving in terms of time and cost. The applicability of FEA in geotechnical engineering can be witnessed from the prominent bearing capacity studies conducted by various researchers namely Griffiths (1982), Sloan and Randolph (1982), Frydman and Burd (1997), Hoshina et al. (2012), Nguyen et al. (2016) and Pham et al. (2019). In light of analysis abnormality and resulting variability in the material stress-strain relationships very close to the shear failure state (De Borst and Vermeer, 1984), the rigid plastic finite element method was developed by Tamura et al. (1984) to analyze the response of soil structure in the limit state. Previous researches, namely to illustrate Mehdi et al. (2014) have made it clear that the flow rule has considerable effect in the results obtained from bearing capacity analysis. Similarly, some experimental studies conducted by Tatsuoka et al. (1986)

have well depicted the influence of confining stress on material "friction angle ϕ" in the case of frictional materials. The rigid plastic finite element method is eminent in terms of ease of introducing the non-associated flow rule to the material properties for diminishing the effect of dilatancy. Therefore, in order to present the actual failure pattern of soil underneath the foundation upon application of load, the non-linear shear strength model is used in this research in contrast with the conventional Mohr-Coulomb or Drucker-Prager criteria to ascertain the size effect of foundation on the ultimate bearing capacity of intermediate soil. The resemblance in the results obtained from the non-linear RPFEM and the AIJ method indicates that RPFEM is not only suitable for analyzing the soil response in the limit state rather and it also well accounts for the size effect of foundation in bearing capacity. Furthermore, the failure modes obtained for the soil mass portray the unerring contoured distribution of equivalent plastic strain rate and velocity vectors. Therefore, obtained results illustrate that RPFEM using non-linear shear strength parameters can better envisage the ultimate bearing capacity as compared to the ordinary bearing capacity formulas currently in practice.

2 CONSTITUTIVE EQUATION FOR RIGID PLASTIC FINITE ELEMENT ANALYSIS

The rigid plastic finite element method (RPFEM) was initially derived using the concept of upper bound theorem of plasticity theory, which in fact applies the upper bound on the actual limit load to be worked out. Reissner (1924) proposed the rigid plastic constitutive equation and authenticated that the results match well with those by upper bound limit analysis. The rigid plastic finite element analysis technique works well with both the linear and non-linear analysis of soil against the confining stresses. The rigid plastic constitutive equation in respect of Drucker-Prager yielding criteria is presented in the section below. Hoshina et al. (2011) introduced the rigid plastic constitutive equation by applying the constraint on dilatancy condition using the penalty method.

2.1 *Drucker-Prager yield criteria and rigid plastic constitutive equation*

Tamura et al. (1987) proposed the stress-strain rate relationship of Drucker-Prager type frictional materials by assuming that the associated flow rule holds. Drucker-Prager criteria describes the linear relationship between shear stress and normal stress in the limit state through material constants. It can be said that Drucker-Prager criteria is generalization of Mohr-Coulomb failure theory. The yield surface of Drucker-Prager criteria can be written as follows:

$$f(\sigma) = aI_1 + \sqrt{J_2} = b \tag{13}$$

Here in the above equation I_1 denotes the first invariant of stress tensor $\boldsymbol{\sigma}_{ij}$, J_2 is the second invariant of deviatoric stress tensor $\boldsymbol{s}_{ij}$. Moreover, a and b represent the material properties i.e. internal friction and cohesion respectively under plane strain.

$$I_1 = tr(\sigma_{ij}) \tag{14}$$

$$J_2 = \frac{1}{2}\boldsymbol{s}_{ij}\boldsymbol{s}_{ij} \tag{15}$$

$$a = \frac{\tan\phi}{\sqrt{9 + 12\tan^2\phi}} \tag{16}$$

$$b = \frac{3c}{\sqrt{9 + 12\tan^2\phi}} \tag{17}$$

The expression for volumetric strain rate is given below:

$$\dot{\varepsilon}_v = tr(\dot{\varepsilon}) = tr\left(\lambda\left(aI + \frac{s}{2\sqrt{J_2}}\right)\right) = \frac{3a}{\sqrt{3a^2 + 1/2}}\dot{e} \tag{18}$$

Here λ is the intermediate plastic multiplier and $\dot{e}$ represents the equivalent strain rate. The unit and deviatoric stress tensors are shown by $\boldsymbol{I}$ and $\boldsymbol{s}$ respectively. The strain rate $\dot{\varepsilon}$is a perfectly plastic component, which should satisfy the following volumetric constraint condition against the dilation property of soil to be compatible with the Drucker-Prager failure surface:

$$h(\dot{\varepsilon}) = \dot{\varepsilon}_v - \frac{3a}{\sqrt{3a^2 + 1/2}}\dot{e} = \dot{\varepsilon}_v - \eta\dot{e} = 0 \tag{19}$$

The stress vector can be resolved in two component vectors as given below. The first term accounts for the stress vector which is determined for the yielding function while the second component determines the indeterminate stress having direction parallel to the one of the side of conical Drucker-Prager yield surface.

$$\sigma = \sigma^1 + \sigma^2 = \frac{b}{\sqrt{3a^2 + 1/2}}\frac{\dot{\varepsilon}}{\dot{e}} + \beta\left(I - \frac{3a}{\sqrt{3a^2 + 1/2}}\frac{\dot{\varepsilon}}{\dot{e}}\right) \tag{20}$$

Here β is the undetermined stress characteristic which remains unknown or undetermined until the boundary value problem satisfying the volumetric constraint condition is solved.

The particular analysis methodology adopted in this study involves the incorporation of constraint condition on the equivalent strain rate through penalty constant in the constitutive equation. The penalty method was introduced by the Hoshina et al. (2011).

$$\sigma = \frac{b}{\sqrt{3a^2 + 1/2}}\frac{\dot{\varepsilon}}{\dot{e}} + k(\dot{\varepsilon}_v - \eta\dot{e})\left(I - \frac{3a}{\sqrt{3a^2 + 1/2}}\frac{\dot{\varepsilon}}{\dot{e}}\right) \tag{21}$$

The above equation and the finite element method using the concept of upper bound theorem in plasticity as formulated by Tamura et al. (1987) is also given. This methodology is termed as RPFEM in the current research. In the rigid plastic finite element method spurious deformation of finite elements as a result of zero energy modes have been witnessed during the analysis. While, rigid plastic constitutive equation using the penalty constant stabilizes the analysis and hence avoids zero energy modes.

2.2 *Ultimate bearing capacity analysis of footing*

In this research study, the finite element analysis is performed for the strip foundation subjected to centric vertical loading and placed on the uniform soil mass. A set of input shear strength parameters have been used for carrying out the bearing capacity analysis and results have been compared with those obtained by conventional bearing capacity formulas being practiced by the engineering community. The comparison of obtained results with the existing formulations is used to assess the efficacy of the method employed in this study. The strength parameters of foundation are set sufficient enough to be rigid. The boundary conditions are set large enough to simulate an infinite soil mass. The typical finite element mesh, boundary conditions and loading arrangements are shown in the Figure 1.

The ultimate bearing capacity analysis is performed for varied foundation widths i.e. 1, 5, 10, 30 and 50m using a set of shear strength parameters i.e. angle of internal friction ϕ=30°, 40° and cohesive shear strength c=0, 10 and 50 kPa. The obtained results showing the velocity field and equivalent strain rate distribution in case of B=10m at ϕ=30° and c=10 kPa is shown in the Figure 2. The strain rate distribution is shown by the colored contours for values ranging from $\dot{e}_{max}$ to $\dot{e}_{min}$ (0). The relative distribution and magnitude of $\dot{e}$ determines the magnitude of ultimate bearing capacity.

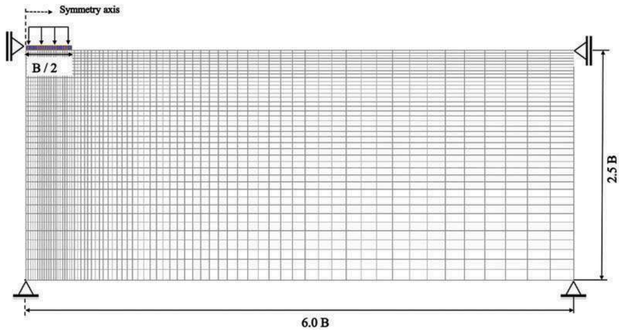

Figure 1. Finite element mesh and boundary condition for foundation width (B=10m).

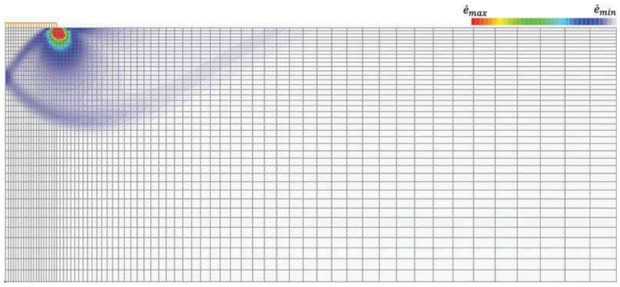

Figure 2. Strain rate distribution for foundation width (B=10m) in case of ϕ=30° and c=10 kPa using Drucker-Prager criteria.

The failure pattern of soil underneath the foundation is similar to that of general failure theories. The maximum horizontal extent of failure mode from the edge of footing is 2.55B and depth is 1.05B with an ultimate bearing capacity of 1955.9 kPa. The failure mode also makes it clear that stress is concentrated on the edge of rigid foundation. This stress concentration seems to depict the problem of singularity in stress distribution, which is addressed by suitable meshing of elements. The efficacy of RPFEM for ultimate bearing capacity analysis is judged by comparing the results with conventional bearing capacity theories. The obtained results have made it clear that in spite of slight singularity in stress the results obtained are well matched with those of the past theories. The bearing capacity results of intermediate soil using Drucker-Prager yield criteria have been obtained for all the cases.

The above graph in Figure 3. shows that ultimate bearing capacity results in case Terzaghi and rigid plastic finite element method with Drucker Prager formulation are close to each other for all foundation widths. But, the results by using AIJ formula are less than others specially in case of larger foundations. As the AIJ formula is based on semi-experimental technique therefore it takes into account the size effect of footing. It infers that RPFEM should be devised in such a way that it can better depict the size effect of foundation in ultimate bearing capacity assessment.

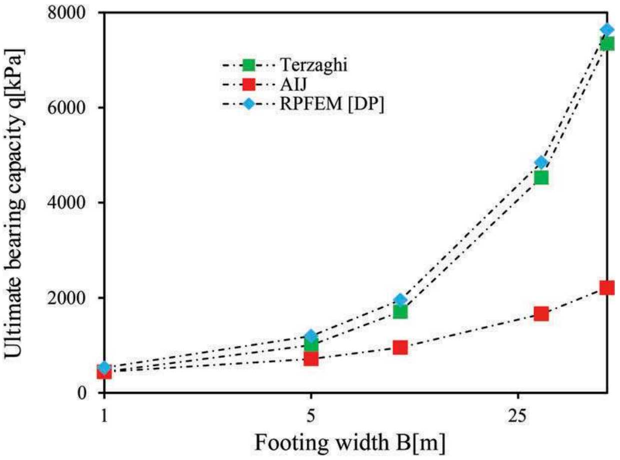

Figure 3. Size effect of foundation on ultimate bearing capacity in case of ϕ=30° and c=10 kPa.

2.3 *Rigid plastic constitutive equation by using non-linear shear strength against confining pressure*

The analysis in this research is based on the following higher order yield function by taking into account the non-linear shear strength of soil.

$$f(\sigma) = aI_1 + (J_2)^n = b \tag{22}$$

Here a and b are the material constants representing the angle of internal friction and cohesion respectively while n depicts the extent of non-linearity in the shear strength of soil against the first stress invariant i.e. I_1. The above equation takes the form of Drucker-Prager yield function for n=1/2. The non-linear parameters have been identified for a series of analyses and comparing the results with AIJ formula which envisage the size effect of foundation. By assuming that the associated flow rule holds, the relationship for the strain rate for non-linear yield function can be expressed as follows:

$$\dot{\varepsilon} = \lambda \frac{\partial f(\sigma)}{\partial(\sigma)} = \lambda \frac{\partial}{\partial(\sigma)}(aI_1 + (J_2)^n - b) = \lambda\left(aI + nJ_2{}^{n-1}s\right) \tag{23}$$

Here λ denotes the plastic multiplier. The strain rate being perfectly plastic component should satisfy the following volumetric constraint condition against the dilation property to figure out the non-linear behavior of soil mass:

$$\dot{\varepsilon}_v = tr\left(\lambda\left(aI + nJ_2{}^{n-1}s\right)\right) = \frac{3a}{\sqrt{3a^2 + 2n^2(b - aI_1)^{2-n^{-1}}}}\dot{e} \tag{24}$$

From Equation (22) and Equation (24) the relationship for the first stress invariant can be easily obtained. The rigid plastic constitutive equation using the non-linear shear strength characteristics against the confining pressure is proposed by Nguyen et al. (2016) as follows:

$$\sigma = \frac{3a}{n}\left[\frac{1}{2n^2}\left\{\left(3a\frac{\dot{e}}{\dot{\varepsilon}_v}\right)^2 - 3a^2\right\}\right]^{\frac{1-n}{2n-1}}\frac{\dot{\varepsilon}}{\dot{\varepsilon}_v} + \left[\begin{array}{l}\frac{b}{3a} - \frac{1}{3a}\left\{\frac{1}{2n^2}\left(3a\frac{\dot{e}}{\dot{\varepsilon}_v}\right)^2 - 3a^2\right\}^{\frac{n}{2n-1}} - \\ \frac{a}{n}\left\{\frac{1}{2n^2}\left(3a\frac{\dot{e}}{\dot{\varepsilon}_v}\right)^2 - 3a^2\right\}^{\frac{1-n}{2n-1}}\end{array}\right] I \tag{25}$$

The value of stress obtained by using the above equation (25) is different from that obtained by using the Drucker-Prager yield function. From Figure 4 it can be seen that non-linear parameter n substantially affects that non-linearity in shear strength against the confining pressure. The ultimate bearing capacity results obtained in Figure 5 by using the multiple non-linear shear strength parameter n with rigid plastic constitutive equation indicates that the results obtained with n=0.54 are well matched with the semi-empirical formula in practice. Nguyen et al. (2016) indicated the

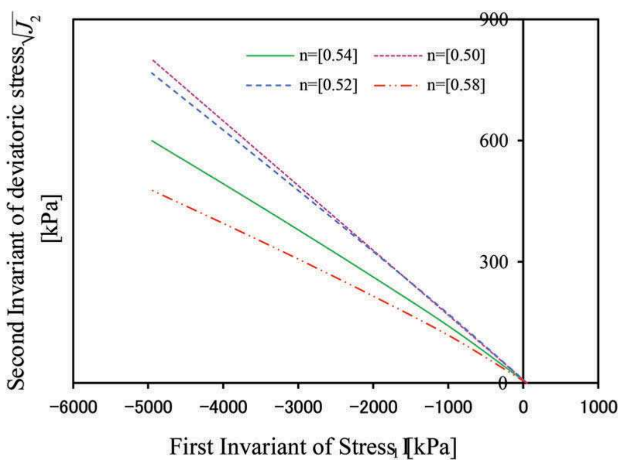

Figure 4. Effect of non-linear parameter n on non-linear shear strength property of soil in case of ϕ=30° and c=10 kPa.

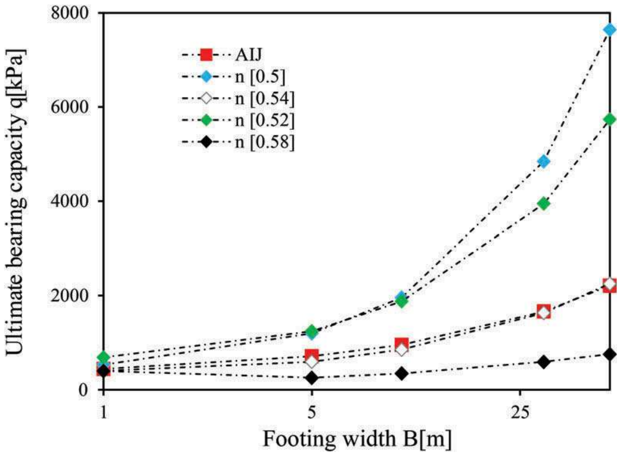

Figure 5. Effect of non-linear parameter n on ultimate bearing capacity of soil in case of ϕ=30° and c=10 kPa.

Table 1. Material characteristics data for analyses.

ϕ	c	a	b	n
	kPa		kPa	
30°	0	0.20	0	0.54
	10	0.20	9.9	0.54
	50	0.21	56	0.54
40°	0	0.25	0	0.525
	10	0.25	7.6	0.525
	50	0.25	43	0.525

coincidence in ultimate bearing capacity between AIJ formula and the computed results by RPFEM employing the non-linear shear strength of sandy soil by using the shear strength property of Toyoura sand. Based on the previous study, this manuscript is attempted to investigate UBC of the intermediate soil within the framework of AIJ formula and the non-linear shear strength of soil is set to fit the AIJ formula. The strength parameters thus obtained presented a good agreement of bearing capacity results with those of AIJ bearing capacity equation in the case of cohesionless soil. Keeping restraint on non-linearity and internal friction angle, the effect of variance in cohesion was then analyzed on the bearing capacity of intermediate soil having given strength characteristics. The results of ultimate bearing capacity obtained by reviewing the non-linearity have been obtained in this way and non-linear parameters a, b and n given in Table 1 have been set for intermediate soil.

3 SIZE EFFECT OF FOUNDATION ON THE ULTIMATE BEARING CAPACITY OF INTERMEDIATE SOIL USING NON-LINEAR SHEAR STRENGTH MODEL

RPFEM by using the Drucker-Prager yield criteria does not estimate the size effect of foundation on the ultimate bearing capacity. The results obtained were quite similar with the conventional bearing capacity theories in practice. The reason behind this fact is that the Drucker-Prager criteria is just generalization of the Mohr-Coulomb failure theory. Therefore, in this study, the rigid plastic finite element method by embedding the non-linear shear strength against the confining stresses has been employed to exactly work out the ultimate bearing capacity of soil underneath the foundation. Previous studies conducted in the purview of critical state soil mechanics have revealed that the peak friction angle does not remain constant with the increase in confining stresses rather than it decreases. This phenomenon of reduction in peak friction angle is attributed to the decrement in dilation caused by high confining stresses.

In the case of ultimate bearing capacity of foundation resting on an infinite soil mass, size of foundation is a factor directly affecting the confining stresses. Therefore, as the footing size increases confining stresses also increase resulting in reduction of peak friction angle. The rigid plastic finite element method with non-linear failure envelope considers the variability in the internal friction angle.

The effect of change in cohesion at the same internal friction angle and non-linearity in shear strength was also analyzed for varied foundation sizes; the typical results of which are shown in Figures 6-8. The failure modes in the case of RPFEM (Higher order) are quite similar to those obtained by RPFEM (Drucker-Prager) but in the case of higher order analysis the area deformed under applied load is smaller than that of linear case. For instance, in the case of Figure 7, B=10m at ϕ=30° and c=10 kPa the size of failure mode from the edge of footing is 1.82B having a depth of 0.88B and the bearing capacity of 852.5 kPa, which is in fact smaller than as computed in Figure 2 using the Drucker-Prager failure theory. Moreover, the obtained results are broadly in well accordance with the AIJ bearing capacity equation implying the efficacy of technique employed. In cohesionless case a small value of cohesion i.e. (c=0.5 kPa) is imparted in analysis to avoid the instability in computation process but the overall effect on results is found to be negligible.

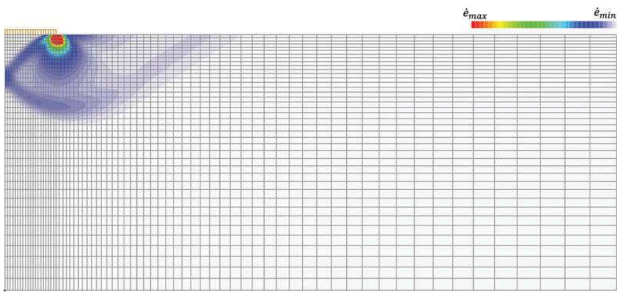

Figure 6. Strain rate distribution for foundation width (B=10m) in case of ϕ=30° and c=0 kPa using RPFEM (Higher order).

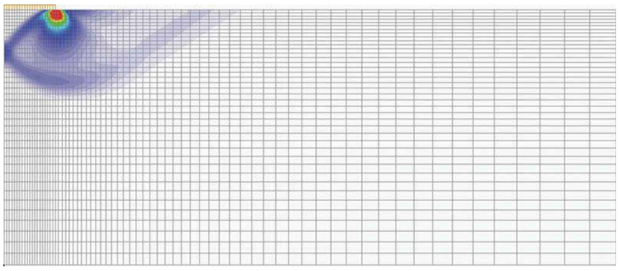

Figure 7. Strain rate distribution for foundation width (B=10m) in case of ϕ=30° and c=10 kPa using RPFEM (Higher order).

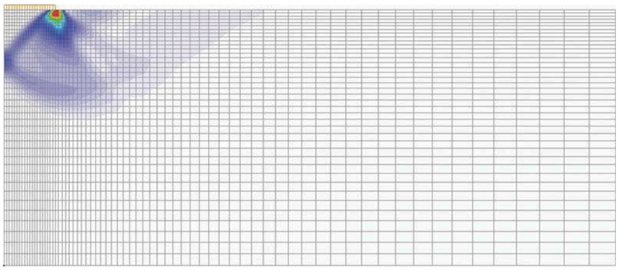

Figure 8. Strain rate distribution for foundation width (B=10m) in case of ϕ=30° and c=50 kPa using RPFEM (Higher order).

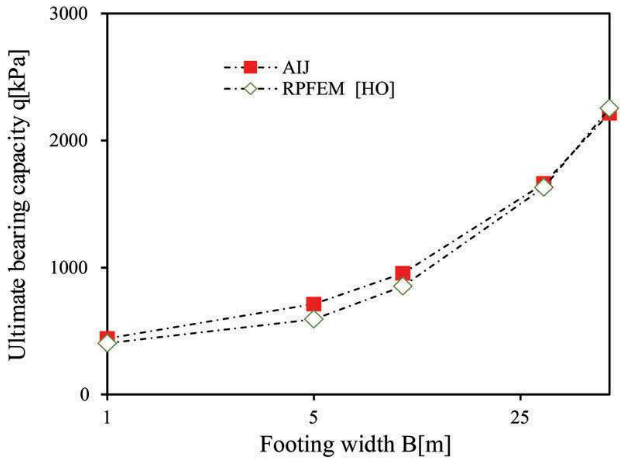

Figure 9. Estimation of UBC in case of ϕ=30° and c=10 kPa.

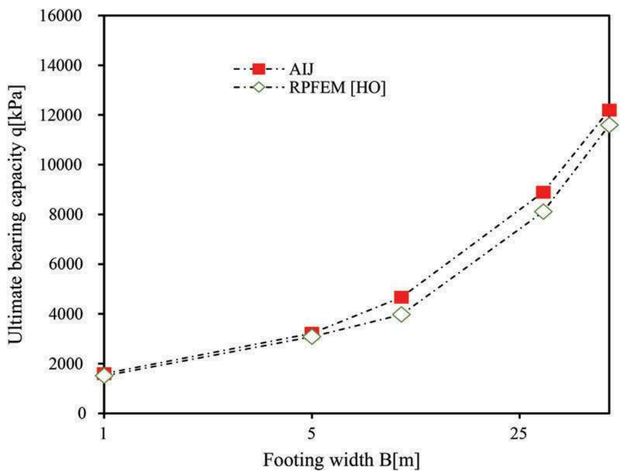

Figure 10. Estimation of UBC in case of ϕ=40° and c=10 kPa.

This study also indicated that upon intrusion of cohesion, the reduction in cohesive shear strength due to confining pressure was also witnessed. It's compound effect on bearing capacity is found to be very small. The intricate non-linear frictional response of soil just below the footing surface can be considered as a logical reason for this decrease in cohesive shear strength. Moreover, this reduction in cohesive shear strength is also a result of particle crushing, negative dilatancy and modified grain size distribution at high confining pressure resulting from large foundation sizes. The mechanical response of soil being investigated depends upon several index physical properties namely to illustrate particle size, shape and hardness. The phenomenon can be better studied by discussing the particle size, shape, crushing index and void ratio before as well as after the tri-axial tests on soil specimens, which is beyond the scope of this study. Although the effect of confining pressure resulting from large foundation sizes on cohesive shear strength of intermediate soils is found to be very small but still considering that effect a modified size effect factor is proposed in the AIJ bearing capacity equation. The size effect factor is calculated from the dispersion of finite element analysis results from the AIJ bearing capacity equation in relation to cohesive shear strength.

The representative bearing capacity results in the Figures 9-10 obtained by non-linear RPFEM are well matched with the AIJ with small discrepancies which in this study is attributed to the size effect of foundation on cohesive shear strength of soil. This effect on cohesive shear strength becomes conspicuous in the case of intermediate soils possessing higher internal friction angle. Therefore, a size effect factor is proposed in the cohesive part of the AIJ ultimate bearing capacity equation. The proposed equation better represents the behavior of intermediate soils which engineers usually come across in practical circumstances. Moreover, the reduction in ultimate bearing capacity due to the effect of confining stresses on cohesive shear strength produces conservative results.

In case of centric vertical load and absence of surcharge the AIJ ultimate bearing capacity equation given in Equation (11) can be rewritten as follows:

$$q = \alpha\eta_c c N_c + \gamma\beta B\left(\frac{B}{B_o}\right)^{\frac{-1}{3}} N_\gamma \qquad (26)$$

From the analysis results, the size effect factor η_c is computed as given below. The value of m= -1/14 is calculated from the obtained results. Hence, the final equation takes the following form:

$$\eta_c = \left(\frac{B}{B_o}\right)^m$$

$B_0 =$ 1m(Reference value in the footing width)

$$q = \left(\frac{B}{B_o}\right)^{\frac{-1}{14}} cN_c + 0.5\gamma B\left(\frac{B}{B_o}\right)^{\frac{-1}{3}} N_\gamma \qquad (27)$$

The typical results reaped from proposed equation are plotted in comparison with the RPFEM (HO) analysis and AIJ results in Figure 11-12:

The exponential factor for the size effect term in adhesion part of UBC equation is obtained through

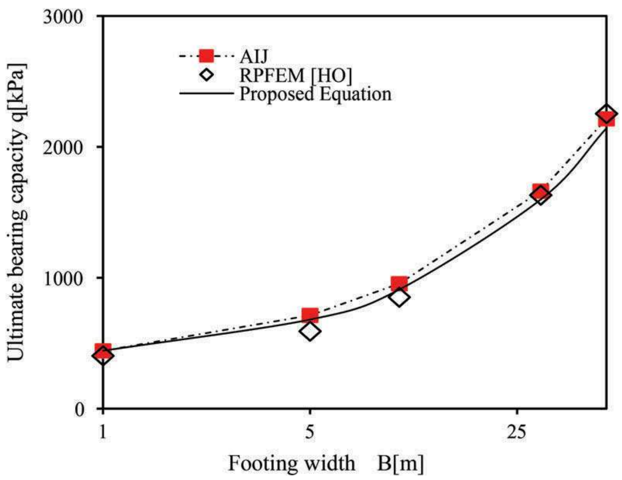

Figure 11. Comparison of results obtained from RPFEM (HO) and proposed equation (27) in case of ϕ=30° and c=10 kPa.

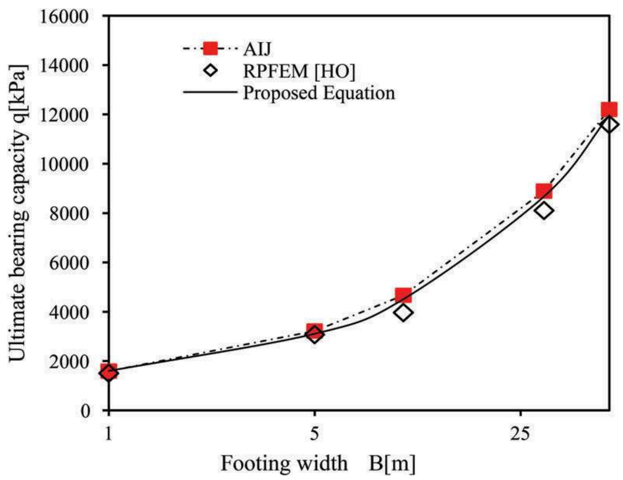

Figure 12. Comparison of results obtained from RPFEM (HO) and proposed equation (27) in case of ϕ=40° and c=10 kPa.

rigorous trial and error mathematical computations based on complete set of analysis results. The obtained correction factor shows that there is a marginal effect of footing size on the cohesive shear strength of intermediate soil. The errors in obtained results remain within 3% on conservative side. The results fetched by using Equation (27) express that the proposed equation better represents the RPFEM analysis results using non-linear shear strength parameters against the confining stresses. It implies that the size effect of foundation also slightly governs the cohesive shear strength in case of intermediate soil. The applicability of Equation (27) is limited to intermediate soil only, as in the case of pure cohesive soil size effect is not observed due to absence of relative frictional mechanism between fines and granular soil which otherwise dominates in intermediate soil.

4 CONCLUSION

The conventional bearing capacity formulas being used by engineering community have very limited applicability due to a couple of disadvantages integral to the theories on the basis of which formulation is done. RPFEM is convenient in its use for analyzing the footing soil system because of its flexibility to employ in multiplex situations in terms of soil strata and footing shapes. The conclusions of this study are recapitulated as follows:

(1) RPFEM has well analyzed the footing soil system in the case of intermediate soils by using non-linear shear strength against the confining stresses.
(2) The results obtained from non-linear RPFEM better estimate the size effect of footing on the UBC.
(3) This study clarified that intermediate soils also have nearly the similar effect of footing size on bearing capacity as that of cohesionless soils.
(4) RPFEM using non-linear shear strength clarified the effect of footing size on cohesive shear strength of intermediate soils.
(5) The effect of non-linearity in shear strength of intermediate soils is envisaged by working out the relationship between the first stress invariant and the second invariant of deviatoric stress.
(6) The effectiveness of RPFEM for UBC was assessed through a set of footing widths, internal friction angle and cohesive shear strength of soil.
(7) The depth of failure modes is interestingly found to be nearly proportionate to footing size.
(8) The broad effectiveness of the AIJ UBC formula is confirmed as discrepancies are minimal.
(9) A thorough investigation of material physical properties before and after the strength tests is necessary to better predict the mechanical properties and their effect on the shear strength.

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